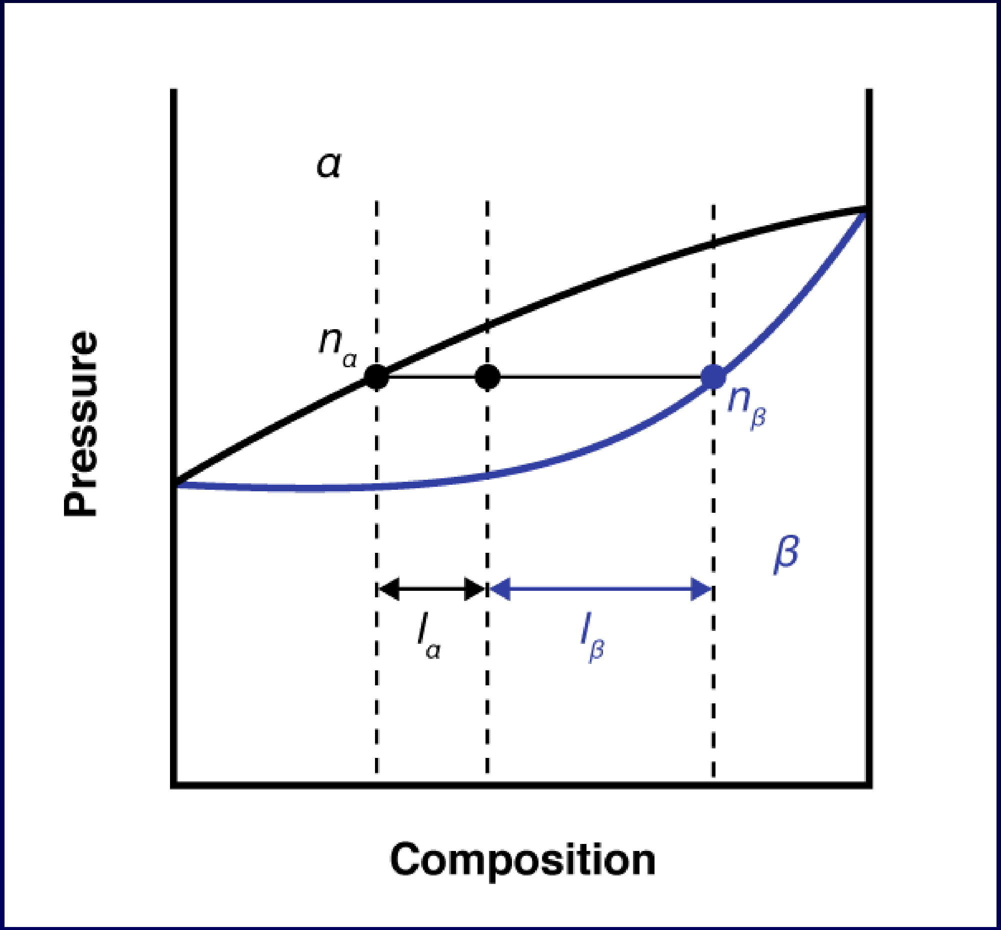
Calculations with Tie Line

What is a tie line: (It is also known as the lever rule)



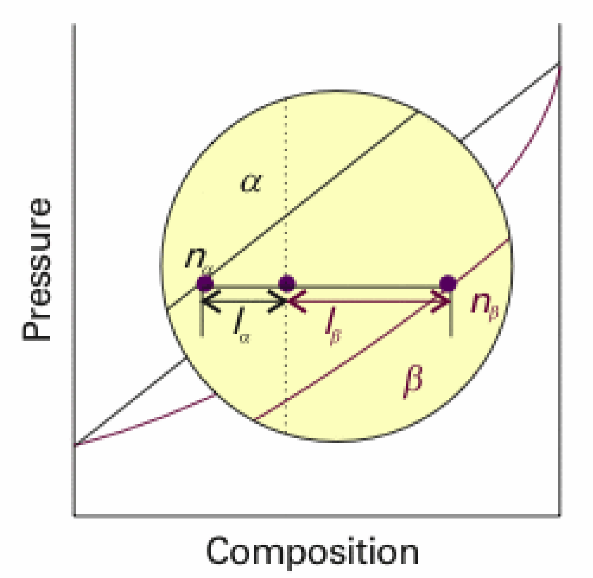
Using the Tie line (Lever Rule)

- Using the **lever rule** one can determine quantitatively the **relative** composition of a **mixture** in a **two-phase** region in a phase diagram. The distances l from the mixture point along a **horizontal** tie line to both phase boundaries give the composition:

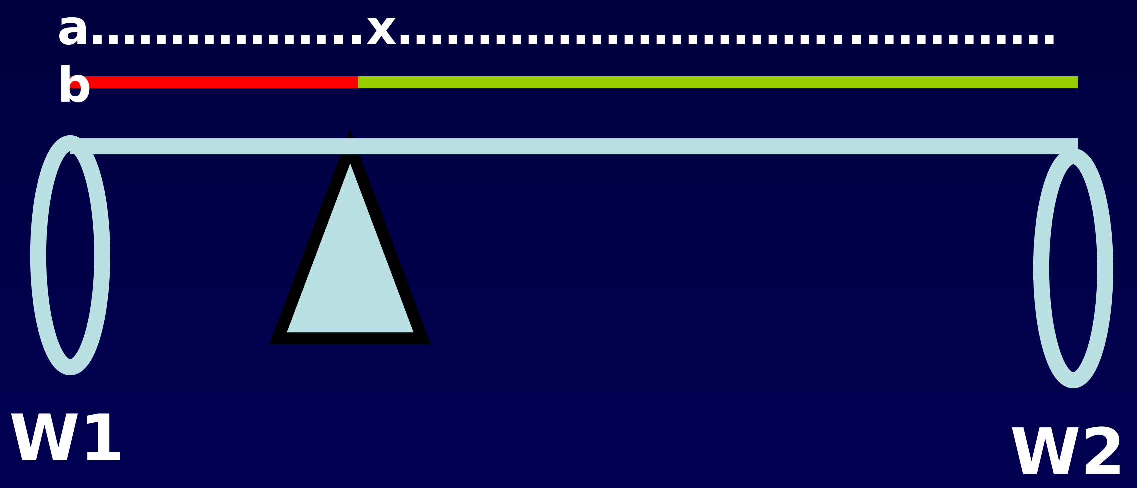
$$n_\alpha \cdot l_\alpha = n_\beta l_\beta$$

n_α represents the amount of phase α and n_β represents the amount of phase β.

- A point in the two-phase region of a phase diagram indicates not only qualitatively that both liquid and vapour are present, but represents quantitatively ***the relative amounts of each***. To find the relative amounts of two phases (alpha) and *(beta)* that are in equilibrium, we measur l_α he l_β distances and along the horizo al tie line, and then use the **le**

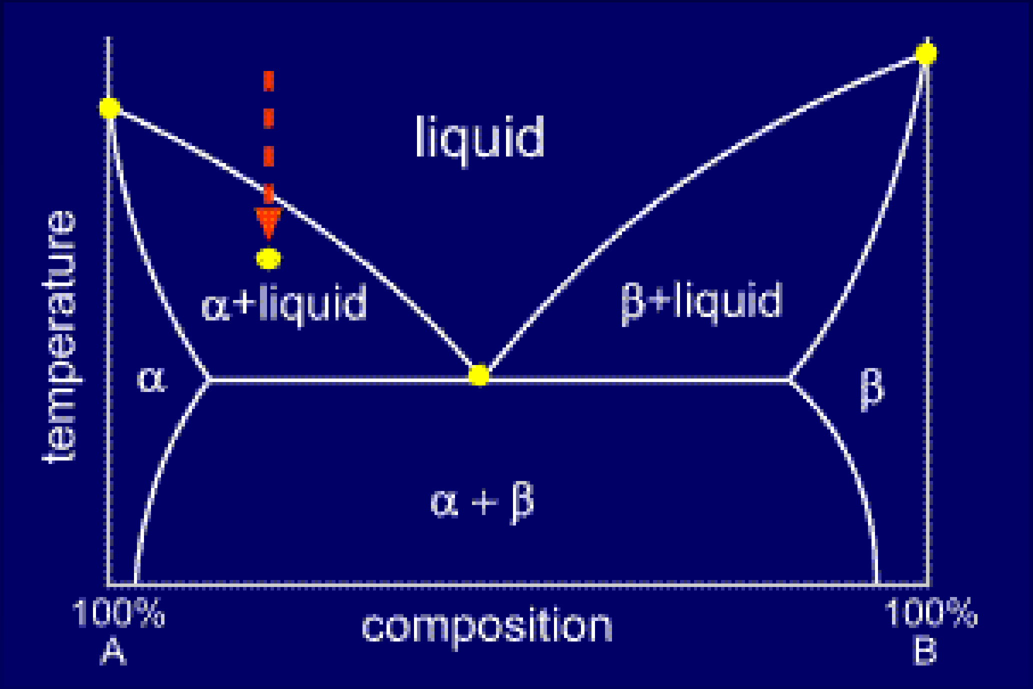


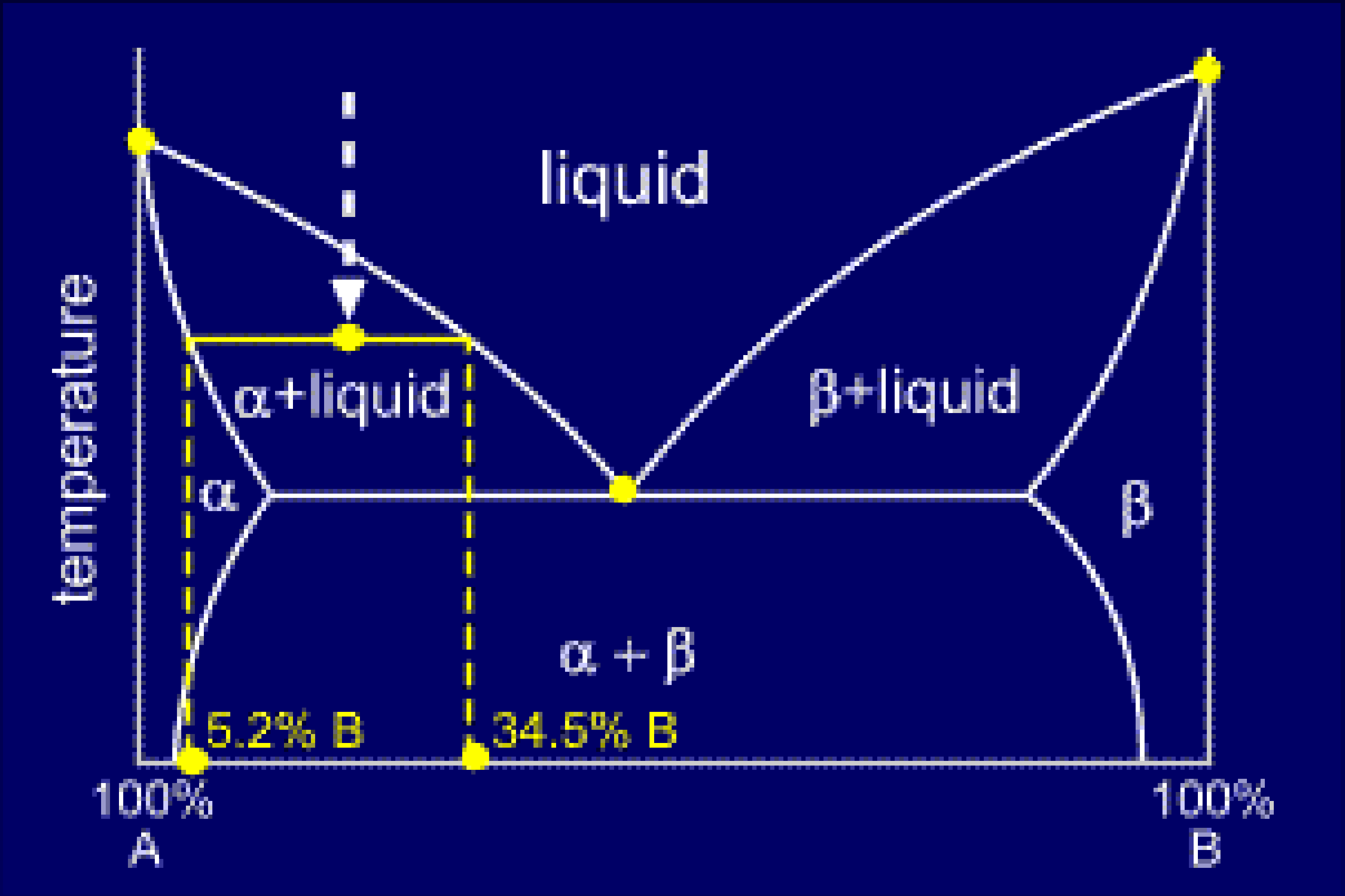
- The lever rule. The distances $l\alpha$ and $l\beta$ are used to find the proportions of the amounts of phases α (such as vapour) and β (for example, liquid) present at equilibrium. The lever rule is so called because a similar rule relates the masses at two ends of a lever to their distances from a pivot ($m\alpha l\alpha = m\beta l\beta$ for balance).



W1 x ax = W2 x bx

- Consider a cooling alloy at the composition and temperature marked on the diagram. As shown on the phase diagram, the alloy is, at the given temperature, a mixture of alpha and liquid phases - but what are their exact compositions at this temperature?

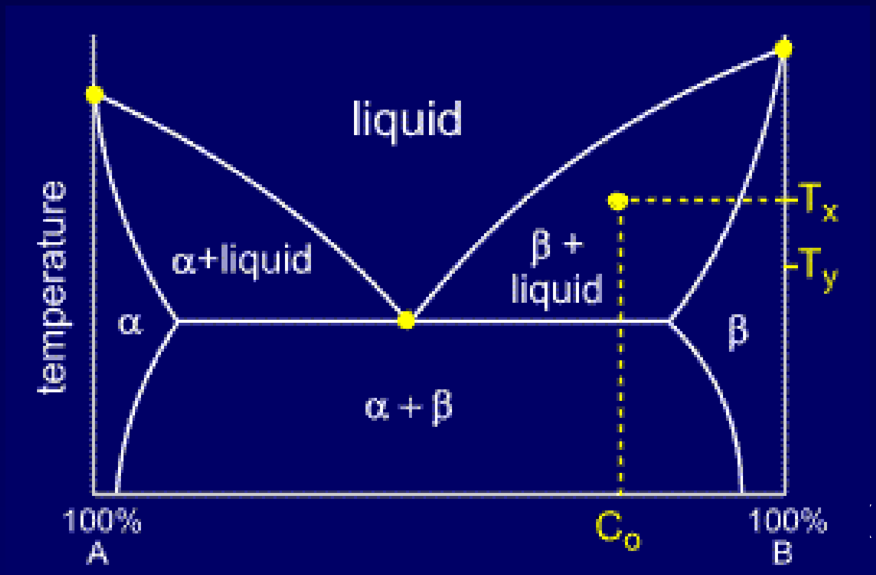



liquid
α+liquid
β+liquid
α
β
α + β
temperature
5.2% B
34.5% B
100%
A
100%
B

- An **isothermal** (constant temperature) line through the alloy's position on the phase diagram when it is in a two phase field, intersecting the two adjacent solubility curves, is called a tie line (yes, that's the horizontal yellow line on the diagram). The ends of the tie lines show the compositions of the two phases that exist in equilibrium with each other ***at this temperature***. From the diagram we know that alpha and liquid phases will exist. The tie line shows that the alpha phase is ***5.2%B*** and the liquid phase is ***34.5%B*** at this temperature.
- Remember, though, that the overall composition of the sample is ***unchanged*** - we are only discovering the ***compositions*** of the constituent phases within the sample.

- For a cooling alloy at composition ***Co*** and temperature ***Tx***, tie lines may be used to answer questions such as:

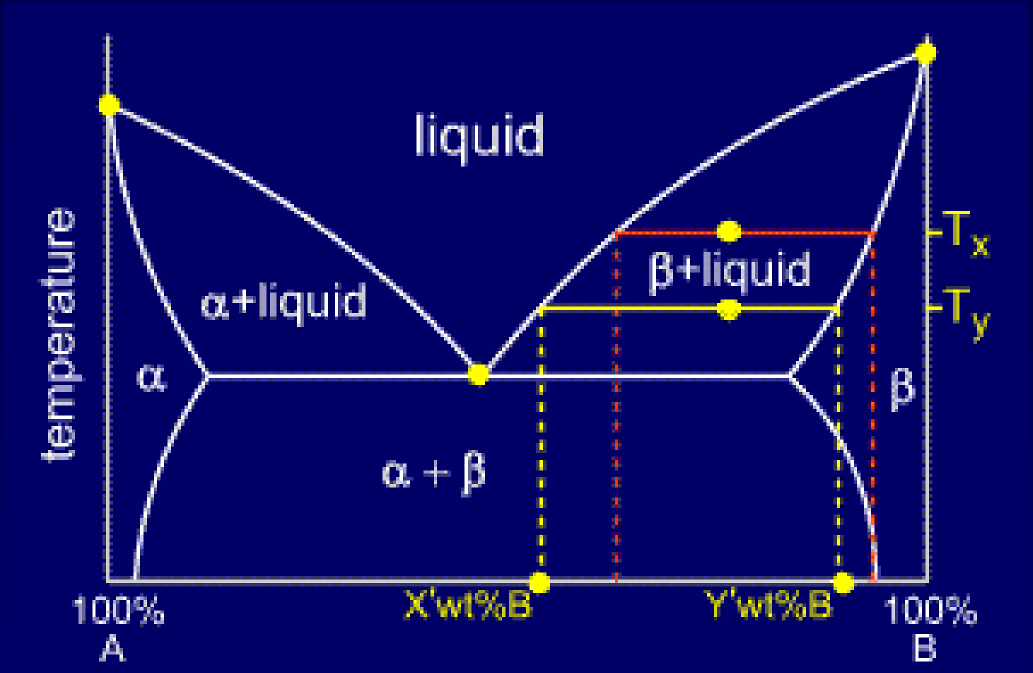
- what phases are present ?
- what are their compositions ?
- if the temperature is reduced to ***Ty***, how do the compositions of the two phases vary ?
- The answer to *"what phases are present ?"* is easy. Composition ***Co*** and temperature ***Tx*** meet in the ***beta + liquid*** phase field, so these are the two phases present



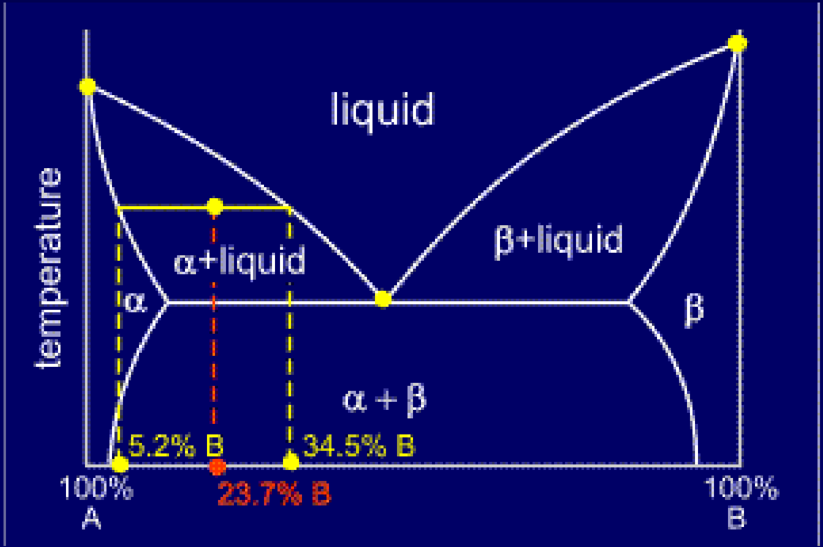
To answer *"what are their compositions ?"* we must draw a horizontal tie line from the point to the nearest phase diagram boundaries. The tie line shows us that the compositions are:

Liquid: *X wt% B*

- To answer the last question *"if the temperature is reduced to Ty, how do the compositions of the two phases vary?"* consider the new tie-line, shown in yellow on the diagram. The compositions of liquid and beta phase have both decreased in wt%B to:
- Liquid: ***X' wt% B***
- Beta: ***Y' wt% B***
- Thus, both the liquid and the beta phases are getting richer in A as the sample is cooled.



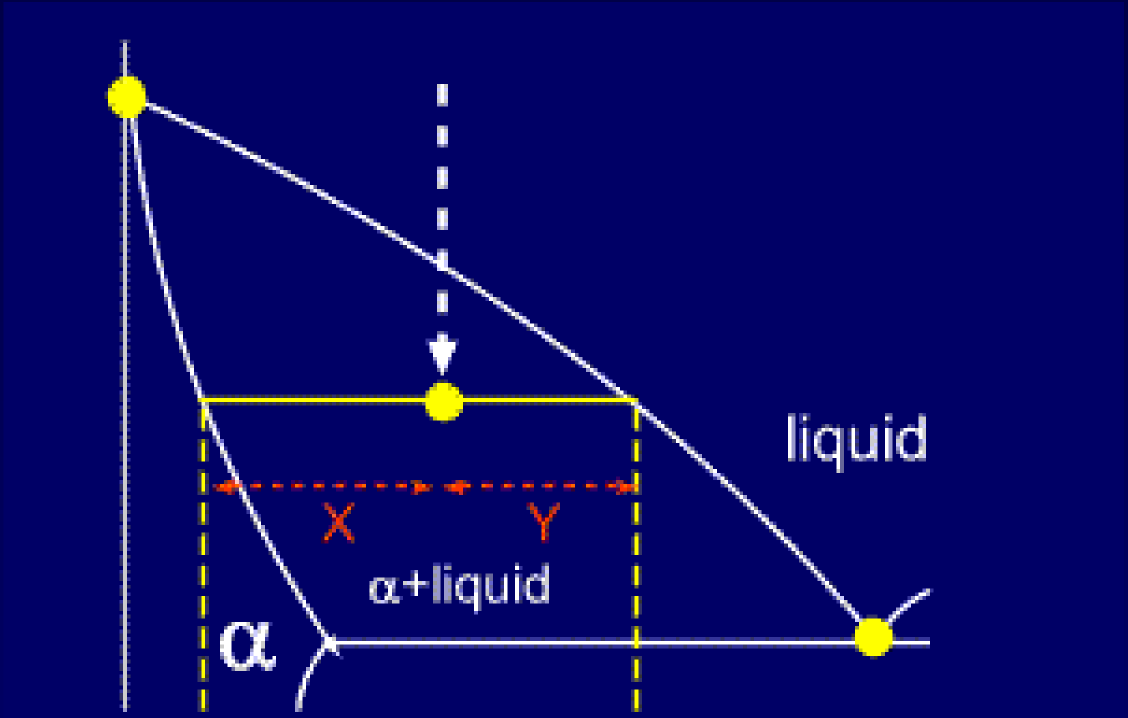
- Distances along the tie line can be found very simply by using a ruler on an ***accurate*** phase diagram or, more correctly, by using data from the composition axis (the x-axis). For example, on the diagram shown, the percentage of alpha present can be calculated from the three pieces of composition data given:
- Fraction of alpha = (34.5 - 23.7) / (34.5 - 5.2) = 0.3686
- Thus, percentage of alpha = 0.3686 x 100 = 36.86%
- and, as the alpha and the liquid make up 100% of the alloy's composition:
- Percentage of liquid = 100 - 36.86 = 63.14%



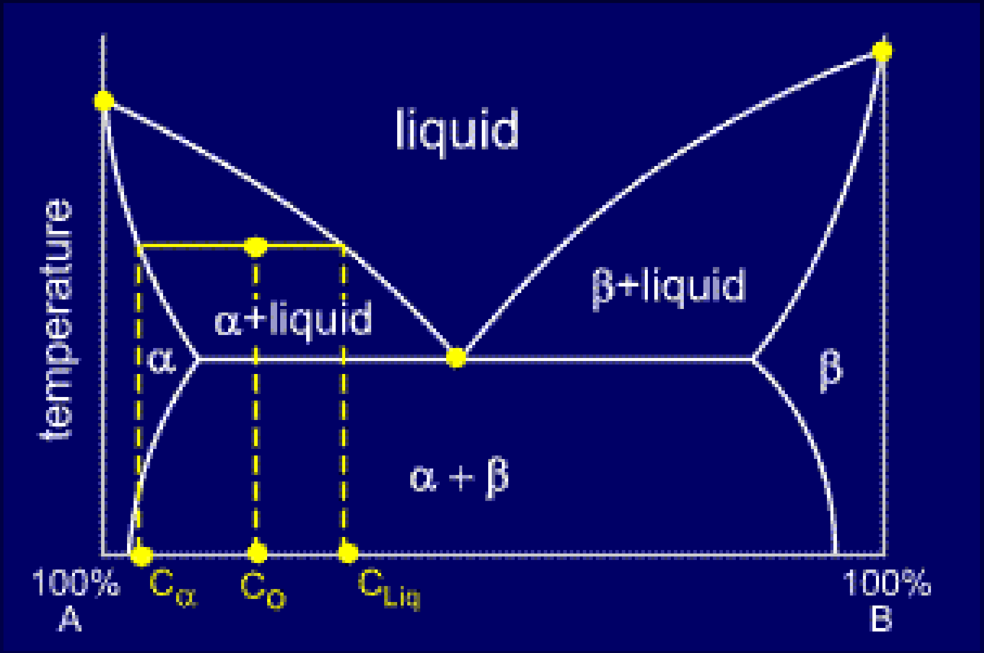
- Basically, the proportions of the phases present are given by the ***relative lengths*** of the tie line. So, the proportions of alpha and liquid present on the diagram (showing a portion of the whole phase diag

$$\frac{x}{x+y}\ and\ \frac{y}{x+y}$$

- Simple, isn't it ?
- But... which equation corresponds to which phase ?



- Now that we know the ***compositions*** of the two phases, we need to find ***how much*** of each phase exists at the given temperature. The ***ratio*** of the two phases present can be found by using the lever rule. At first sight the lever rule can appear confusing. It is really invoking the conservation of mass, and can be proved mathematically, as shown below the diagram.
- Essentially, we start off with an overall composition of our alloy - Co. From the tie-line we know that the two phases at a given temperature have two different compositions, but overall the amounts of these two compositions must add up to the alloy's overall composition, Co.
- This is the basis for the lever rule. Using the lever rule itself is very simple, we'll show you



C_o must be made up of appropriate amounts of α at composition C_α and of liquid at composition C_{Liq}.

where f_α is the fraction of α in the sample:

$$C_o = f_\alpha C_\alpha + (1 - f_\alpha) C_{Liq}$$

and by rearranging:

$$C_o - C_{Liq} = f_\alpha (C_\alpha - C_{Liq})$$

and finally:

$$\boxed{f_\alpha = \frac{C_o - C_{Liq}}{C_\alpha - C_{Liq}}}$$